To honor Davis & Pérusse and repeal their glossary of processes of numerical competence

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I commend Davis & Pérusse (D & P) for tackling the "definitional confusion arising from inconsistent terminology" but disagree with their result summarized in Table 1. I propose an alternative set of processes that I believe is more precise, logical, consistent, and complete. I then comment as space permits on points of agreement or disagreement with D & P.

To summarize, I introduce *absolute numerousness judgments* (ANJs) which together with relative numerousness judgments (RNJs) and counting comprise the three basic processes of numerical competence up to the level of counting. I distinguish between (a) *precise* and *imprecise* ANJs and (b) the use of *nonsymbolic* and *symbolic* processes in both ANJs and RNJs. I am then able to delete D & P's "subitizing," "estimation," "protocounting," and "concept of number" without omitting any necessary processes of numerical competence. My glossary was intended to address only numerical competence involving entities, but I recognize that it applies to temporal events as well.

Glossary of numerical competence processes. A necessary prerequisite to the identification of numerical processes is Stevens's distinction between "numerousness" and "numerosity." Numerousness is "the cardinality attribute of groups of object . . . that we observe when we look at, but do not count, a collection of objects" (1951, p. 22) and numerosity is the cardinality attribute that is determined by counting.

Absolute numerousness judgments (ANJs). Such judgments are absolute in the sense of being self-sufficient or unrelated to anything else; for example, responding to "threeness" as opposed to "twoness" or "fourness" (e.g., Hicks 1956) or using a numeral or any other symbol to represent the cardinality attribute of any set of three objects. ANJs can be *precise* or *imprecise* and can involve *nonsymbolic* or *symbolic* processes. *Precise ANJs* can be made (a) nonsymbolically using discrimination tasks where immediately successive numbers of entities constitute the discriminanda or, (b) symbolically using numerals or other symbols. Symbols may be used in conjunction with (a) the stimulus as a conditional cue which specifies the correct choice, (b) the response as a means to designate the cardinality attribute of the group of objects, or (c) both. The use of symbols assumes prerequisite training and experience. Precise ANJs replace D & P's "subitizing" (see below). *Imprecise ANJs* differ only in being less precise; they replace D & P's "estimation" (see below).

Relative numerousness judgments (RNJs). Such judgments are evident in categorical responses to discriminanda in terms of "more," "fewer," "intermediate," and so on, numbers of entities. These can be investigated nonsymbolically, that is, always reinforcing responses to the discrimination which manifests the "more," "less," or "intermediate" number of entities. The category required on a given trial can be cued symbolically, e.g., Dooley and Gill's (1977) use of lexigrams or Thomas and Chase's (1980) use of one ("fewer"), two ("intermediate"), or three ("more") cue lights. Although responses could be made symbolically, it is difficult to imagine why symbolic responses would be better than merely touching (or in some way "pointing to") the discriminandum that manifests the correct choice.

Counting. I agree with D & P's use of Gelman and Gallistel's (1978) five principles of counting as the criteria for counting, and with D & P's equivocation about the need for the fourth and fifth principles.

Davis & Pérusse's nomenclature. "Subitizing" was deleted because (a) D & P, as shown below, have changed its meaning too much from Kaufman et al.'s (1949) meaning and because (b) "precise ANJs" describe more effectively a process that is most akin to subitizing and most likely to be amenable to research using animals.

Kaufman et al. (1949) concluded, "On the average, *subitizing* is a considerably more accurate, more rapid, and more confident process than *estimating*" (p. 525). I agree with D & P that measures of confidence with nonverbal animals may be too problematic, but I no longer agree that measures of speed are of "minor relevance to work with noncounting subjects." The ability to count is not relevant; competence in using symbols in conjunction with ANJs is sufficient. By eliminating confidence and speed as criteria, D & P abandoned subitizing as Kaufman et al. defined it. The study of subitizing remains useful, but finding ways to contrast subitizing and estimation as Kaufman et al. did may make such studies with animals too formidable to attempt.

My "imprecise ANJs" using symbols seems to be a more definitive version of D & P's "enhanced" estimation. D & P go on to make the unnecessary assumption that enhanced estimation is a "postcounting process," an assumption that also seems inconsistent with their saying that enhanced estimation is done "without counting" (Table 1).

"Protocounting" is unjustified and unjustifiable. D & P's definition assumes (a) that other numerical processes such as "relative numerousness judgments and subitizing have been precluded" and (b) that "control tests . . . have not yet revealed evidence of 'true' counting" (Table 1). Methodologically, the latter makes no sense as a criterion; and even if it made sense, how could the possible involvement of other numerical processes be precluded? If counting cannot be demonstrated and if the performance involves numerical competence, then one of the other processes can presumably be demonstrated.

"Concept of number" as a separate process is superfluous. Any valid study of numerical competence must involve conceptual processes. Otherwise, it may only involve rote memorization of particular exemplars where the "numerical" aspect of the stimuli is not essential. The basic issue here is discussed more fully (in a different context) in Thomas and Noble (1988). D & P's "concept of number" is reducible to the subject's ability to *generalize* the use of ANJs, RNJs, and counting.

On a minor point, D & P appear to criticize Thomas et al. (1980) for failing to make a stronger case for RNJs. We tried (successfully, I believe) to establish the squirrel monkey's capacity for precise numerousness discriminations; to facilitate this we allowed both ANJ and RNJ solutions to occur. We discussed the design and results as they related to both solutions and noted that the evidence suggests one of the monkeys responded on a relative basis.

Thomas and Chase (1980) provided evidence for RNJs by squirrel monkeys in an article not cited by D & P. This is a significant omission from their section on ordinality, because they cited Thomas and Ingram's (1979) study of ordinal volume judgments by squirrel monkeys as one to be excluded from discussion because it was "not primarily numerical." The Thomas and Chase study was numerical and may have been the first to show evidence for ordinal numerousness judgments by nonhuman animals.

Finally, some will have observed that my title imitates (flattery intended) Stevens's (1961) "To Honor Fechner and Repeal His Law." So far, the honor due D & P has been overshadowed. I respect them for grappling with this complex and chaotic literature and for doing so in such an intensely critical forum.